A **polygon** is a closed plane figure formed by **three** or more segments that intersect only at their endpoints. Each segment that forms a polygon is a **side** of the polygon. The **common** endpoint of two **sides** is a **vertex** of the polygon. A segment that connects any two **nonconsecutive** vertices is a **diagonal**.

You can name a polygon by the number of its sides. The table shows the names of some common polygons.

<table>
<thead>
<tr>
<th>Number of Sides</th>
<th>Name of Polygon</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>triangle</td>
</tr>
<tr>
<td>4</td>
<td>quadrilateral</td>
</tr>
<tr>
<td>5</td>
<td>pentagon</td>
</tr>
<tr>
<td>6</td>
<td>hexagon</td>
</tr>
<tr>
<td>7</td>
<td>heptagon</td>
</tr>
<tr>
<td>8</td>
<td>octagon</td>
</tr>
<tr>
<td>9</td>
<td>nonagon</td>
</tr>
<tr>
<td>10</td>
<td>decagon</td>
</tr>
<tr>
<td>12</td>
<td>dodecagon</td>
</tr>
<tr>
<td>(n)</td>
<td>(n)-gon</td>
</tr>
</tbody>
</table>

If the number of sides is not listed in the table, then the polygon is called a \(n\)-gon where \(n\) represents the number of sides.

Example: a 16 sided figure is called a 16-gon.

All the sides are congruent in an **equilateral** polygon. All the angles are congruent in an **equiangular** polygon. A **regular** polygon is one that is both equilateral and equiangular. If a polygon is not regular, it is called **irregular**.

A polygon is **concave** if any part of a **diagonal** contains points in the **exterior** of the polygon.
Chapter 6 Study Guide Answers

If no __diagonals__ contains points in the __exterior__, then the polygon is __convex__. A __regular__ polygon is always convex.

To find the sum of the __interior__ angle measures of a __convex__ polygon, draw all possible __diagonals__ from one vertex of the polygon. This creates a set of __triangles__. The sum of the angle measures of all the triangles __equals__ the sum of the angle measures of the polygon.

By the Triangle Sum Theorem, the sum of the interior angle measures of a triangle is 180°.

<table>
<thead>
<tr>
<th>Polygon</th>
<th>Number of Sides</th>
<th>Number of Triangles</th>
<th>Sum of Interior Angle Measures</th>
</tr>
</thead>
<tbody>
<tr>
<td>Triangle</td>
<td>3</td>
<td>1</td>
<td>(1) 180° = 180°</td>
</tr>
<tr>
<td>Quadrilateral</td>
<td>4</td>
<td>2</td>
<td>(2) 180° = 360°</td>
</tr>
<tr>
<td>Pentagon</td>
<td>5</td>
<td>3</td>
<td>(3) 180° = 540°</td>
</tr>
<tr>
<td>Hexagon</td>
<td>6</td>
<td>4</td>
<td>(4) 180° = 720°</td>
</tr>
<tr>
<td>n-gon</td>
<td>n</td>
<td>n - 2</td>
<td>(n - 2) 180°</td>
</tr>
</tbody>
</table>

In each __convex__ polygon, the number of triangles formed is __two__ less than the number of __sides__ n. So the __sum__ of the angle measures of all these triangles is (n − 2)180°.

**Theorem 6.1-1**  Polygon Angle Sum Theorem

The sum of the interior angle measures of a convex polygon with n sides is (n − 2)180°.

An __exterior__ angle is formed by one side of a polygon and the extension of a consecutive side.

**Theorem 6.1-2**  Polygon Exterior Angle Sum Theorem

The sum of the exterior angle measures, one angle at each vertex, of a convex polygon is 360°.
### Chapter 6 Study Guide Answers

#### In 1-6, tell whether the figure is a polygon. If it is a polygon, name it by the number of its sides.

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td><img src="image" alt="Hexagon" /></td>
<td>polygon hexagon</td>
</tr>
<tr>
<td>2.</td>
<td><img src="image" alt="Heptagon" /></td>
<td>polygon heptagon</td>
</tr>
<tr>
<td>3.</td>
<td><img src="image" alt="Not a Polygon" /></td>
<td>not a polygon</td>
</tr>
<tr>
<td>4.</td>
<td><img src="image" alt="Not a Polygon" /></td>
<td>not a polygon</td>
</tr>
<tr>
<td>5.</td>
<td><img src="image" alt="Nonagon" /></td>
<td>polygon nonagon</td>
</tr>
<tr>
<td>6.</td>
<td><img src="image" alt="Not a Polygon" /></td>
<td>not a polygon</td>
</tr>
</tbody>
</table>

#### In 7-12, tell whether the polygon is regular or irregular. Tell whether it is concave or convex.

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>7.</td>
<td><img src="image" alt="Irregular Convex" /></td>
<td>irregular convex</td>
</tr>
<tr>
<td>8.</td>
<td><img src="image" alt="Irregular Concave" /></td>
<td>irregular concave</td>
</tr>
<tr>
<td>9.</td>
<td><img src="image" alt="Regular Convex" /></td>
<td>regular convex</td>
</tr>
<tr>
<td>10.</td>
<td><img src="image" alt="Regular Convex" /></td>
<td>regular convex</td>
</tr>
<tr>
<td>11.</td>
<td><img src="image" alt="Irregular Concave" /></td>
<td>irregular concave</td>
</tr>
<tr>
<td>12.</td>
<td><img src="image" alt="Irregular Concave" /></td>
<td>irregular concave</td>
</tr>
</tbody>
</table>
Chapter 6 Study Guide Answers

<table>
<thead>
<tr>
<th>Problem 13</th>
<th>Find the sum of the interior angle measures of a convex heptagon.</th>
</tr>
</thead>
<tbody>
<tr>
<td>((n-2)180^\circ)</td>
<td>((7-2)180^\circ) = (900^\circ)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Problem 14</th>
<th>Find the measure of each interior angle of a regular decagon.</th>
</tr>
</thead>
<tbody>
<tr>
<td>((n-2)180^\circ)</td>
<td>((10-2)180^\circ) = (1440^\circ) (\frac{1440^\circ}{10} = 144^\circ)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Problem 15</th>
<th>Find the measure of each interior angle of pentagon ABCDE.</th>
</tr>
</thead>
<tbody>
<tr>
<td>((5-2)180^\circ) = (540^\circ)</td>
<td>(m\angle A + m\angle B + m\angle C + m\angle D + m\angle E = 540^\circ)</td>
</tr>
<tr>
<td>(35c + 18c + 32c + 32c + 18c = 540)</td>
<td>(135c = 540)</td>
</tr>
<tr>
<td>(\frac{135}{135}c = 4)</td>
<td>(m\angle A = 35(4) = 140^\circ)</td>
</tr>
<tr>
<td>(m\angle B = m\angle E = 18(4) = 72^\circ)</td>
<td>(m\angle C = m\angle D = 32(4) = 128^\circ)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Problem 16</th>
<th>The measure of each interior angle of a regular polygon is (135^\circ). How many sides does the polygon have?</th>
</tr>
</thead>
<tbody>
<tr>
<td>(n) ((n-2)180^\circ = 135^\circ)</td>
<td>(n) ((n-2)180 = 135n)</td>
</tr>
<tr>
<td>(180n - 360 = 135n)</td>
<td>(-180n) (-180n)</td>
</tr>
<tr>
<td>(-360 = -45n)</td>
<td>(-45) (-45)</td>
</tr>
<tr>
<td>(n = 8)</td>
<td>()</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Problem 17</th>
<th>Find the measure of each exterior angle of a regular dodecagon.</th>
</tr>
</thead>
<tbody>
<tr>
<td>A dodecagon has 12 sides.</td>
<td>sum of ext. (\angle s = 360^\circ)</td>
</tr>
<tr>
<td>(360^\circ) (\frac{360^\circ}{12} = 30^\circ)</td>
<td>()</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Problem 18</th>
<th>Find the value of (b) in polygon FGHJKL.</th>
</tr>
</thead>
<tbody>
<tr>
<td>(15b + 18b + 33b + 16b + 10b + 28b = 360)</td>
<td>(120b = 360)</td>
</tr>
<tr>
<td>(\frac{120}{120} = 360)</td>
<td>(b = 3)</td>
</tr>
</tbody>
</table>
Geometry – Section 6.2 – Notes and Examples – Properties of Parallelograms

Any polygon with four sides is a quadrilateral. However, some quadrilaterals have special properties. These special quadrilaterals are given their own names.

A quadrilateral with two pairs of parallel sides is a parallelogram. To write the name of a parallelogram, you use the symbol \( \Box \).

Parallelogram \( ABCD \)
\[ \Box ABCD \]
\[ AB \parallel CD, BC \parallel DA \]

**Theorem 6-2-1** Properties of Parallelograms

<table>
<thead>
<tr>
<th>THEOREM</th>
<th>HYPOTHESIS</th>
<th>CONCLUSION</th>
</tr>
</thead>
<tbody>
<tr>
<td>If a quadrilateral is a parallelogram, then its opposite sides are congruent. ((\Box \rightarrow \text{opp. sides } \cong))</td>
<td>[ \begin{array}{c} AB \cong CD \ BC \cong DA \end{array} ]</td>
<td></td>
</tr>
</tbody>
</table>

Opposite sides of a quadrilateral do not share a vertex. Opposite angles do not share a side.

**Theorems** Properties of Parallelograms

<table>
<thead>
<tr>
<th>THEOREM</th>
<th>HYPOTHESIS</th>
<th>CONCLUSION</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>6-2-2</strong> If a quadrilateral is a parallelogram, then its opposite angles are congruent. ((\Box \rightarrow \text{opp. } \angle \cong))</td>
<td>[ \begin{array}{c} \angle A \cong \angle C \ \angle B \cong \angle D \end{array} ]</td>
<td></td>
</tr>
<tr>
<td><strong>6-2-3</strong> If a quadrilateral is a parallelogram, then its consecutive angles are supplementary. ((\Box \rightarrow \text{cons. } \angle \text{ supp.}))</td>
<td>[ \begin{array}{c} m\angle A + m\angle B = 180^\circ \ m\angle B + m\angle C = 180^\circ \ m\angle C + m\angle D = 180^\circ \ m\angle D + m\angle A = 180^\circ \end{array} ]</td>
<td></td>
</tr>
<tr>
<td><strong>6-2-4</strong> If a quadrilateral is a parallelogram, then its diagonals bisect each other. ((\Box \rightarrow \text{diags. bisect each other}))</td>
<td>[ \begin{array}{c} \overline{AZ} \cong \overline{CZ} \ \overline{BZ} \cong \overline{DZ} \end{array} ]</td>
<td></td>
</tr>
</tbody>
</table>
Chapter 6 Study Guide Answers

Problem 1
In \( \square CDEF, \, DE = 74, \, DG = 31, \) and \( m \angle FCD = 42^\circ \). Find the measures of \( CF, \) \( \angle DEF, \) and \( DF \).

\[
\begin{align*}
CF &= 74 \\
m \angle DEF &= 42^\circ \\
DF &= 2(31) = 62
\end{align*}
\]

Problem 2
\( WXYZ \) is a parallelogram. Find the measures of \( YZ, \) and \( \angle Z \).

\[
\begin{align*}
8a - 4 &= 6a + 10 \\
6a &= 14 \\
a &= 7 \\
YZ &= 8a - 4 = 52 \\
m \angle Z &= 9b + 2 = 65^\circ
\end{align*}
\]

Problem 3
\( EFGH \) is a parallelogram. Find the measures of \( JG \) and \( FH \).

\[
\begin{align*}
EJ &= JG \\
3w &= w + 8 \\
-w &= -w \\
2w &= 8 \\
w &= 4 \\
JG &= w + 8 = 12 \\
FH &= 2(2z) = 2(4.5) = 18
\end{align*}
\]

Problem 4
Three vertices of \( \square JKLM \) are \( J(3, -4), \) \( K(-2, 2), \) and \( L(2, 4). \) Find the coordinates of vertex \( M. \)

\[
\begin{align*}
\text{Opposite sides are parallel} \\
\text{Find slope of } KL \text{ by counting} \\
\text{Over 4 up 2} \\
\text{From point } J, \text{ go over 4 and up 2, this is your point } M. \\
M(7, -2)
\end{align*}
\]
Geometry – Section 6.3 – Notes and Examples – Conditions for Parallelograms

You have learned to identify the properties of a parallelogram. Now you will be given the properties of a quadrilateral and will have to tell if the quadrilateral is a parallelogram. To do this, you can use the definition of a parallelogram or the conditions below.

<table>
<thead>
<tr>
<th>Theorem</th>
<th>Theorem</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>6-3-1</strong></td>
<td>If one pair of opposite sides of a quadrilateral are parallel and congruent, then the quadrilateral is a parallelogram. (quad. with pair of opp. sides (\parallel) and (\equiv))</td>
<td><img src="image" alt="Diagram" /></td>
</tr>
<tr>
<td><strong>6-3-2</strong></td>
<td>If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram. (quad. with opp. sides (\equiv))</td>
<td><img src="image" alt="Diagram" /></td>
</tr>
<tr>
<td><strong>6-3-3</strong></td>
<td>If both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram. (quad. with opp. (\triangle) (\equiv))</td>
<td><img src="image" alt="Diagram" /></td>
</tr>
</tbody>
</table>

The two theorems below can also be used to show that a given quadrilateral is a parallelogram.

<table>
<thead>
<tr>
<th>Theorem</th>
<th>Theorem</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>6-3-4</strong></td>
<td>If an angle of a quadrilateral is supplementary to both of its consecutive angles, then the quadrilateral is a parallelogram. (quad. with (\angle) supp. to cons. (\triangle) (\rightarrow))</td>
<td><img src="image" alt="Diagram" /></td>
</tr>
<tr>
<td><strong>6-3-5</strong></td>
<td>If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram. (quad. with diags. bisecting each other (\rightarrow))</td>
<td><img src="image" alt="Diagram" /></td>
</tr>
</tbody>
</table>
To say that a quadrilateral is a parallelogram by definition, you must show that **both** pairs of opposite sides are parallel.

You have learned several ways to determine whether a quadrilateral is a parallelogram. You can use the given information about a figure to decide which condition is best to apply.

## Conditions for Parallelograms

<table>
<thead>
<tr>
<th>Condition</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Both pairs of opposite sides are parallel. (definition)</td>
<td></td>
</tr>
<tr>
<td>One pair of opposite sides are parallel and congruent. (Theorem 6-3-1)</td>
<td></td>
</tr>
<tr>
<td>Both pairs of opposite sides are congruent. (Theorem 6-3-2)</td>
<td></td>
</tr>
<tr>
<td>Both pairs of opposite angles are congruent. (Theorem 6-3-3)</td>
<td></td>
</tr>
<tr>
<td>One angle is supplementary to both of its consecutive angles. (Theorem 6-3-4)</td>
<td></td>
</tr>
<tr>
<td>The diagonals bisect each other. (Theorem 6-3-5)</td>
<td></td>
</tr>
</tbody>
</table>

To show that a quadrilateral is a parallelogram, you only have to show that it satisfies one of these sets of conditions.

### Problem 1
Show that $JKLM$ is a parallelogram for $a = 3$ and $b = 9$. Find $JK$, $LM$, $KL$, and $JM$.

- $JK = 15a - 11 = 15(3) - 11 = 34$
- $LM = 10a + 4 = 10(3) + 4 = 34$
- $KL = 5b + 6 = 5(9) + 6 = 51$
- $JM = 8b - 21 = 8(9) - 21 = 51$

Since both pairs of opposite sides are congruent, $JKLM$ is a parallelogram.

### Problem 2
Show that $PQRS$ is a parallelogram for $x = 10$ and $y = 6.5$.

- $m\angle Q = (6y + 7)^\circ = (6(6.5) + 7)^\circ = 66^\circ$
- $m\angle S = (8y - 6)^\circ = (8(6.5) - 6)^\circ = 46^\circ$
- $m\angle R = (15x - 16)^\circ = (15(10) - 16)^\circ = 134^\circ$

Since one $\angle$ is supplementary to its consecutive $\angle$s, $PQRS$ is a parallelogram.
Chapter 6 Study Guide Answers

In 3-6, determine if the quadrilateral must be a parallelogram. Justify your answer.

### Problem 3

![Diagram of a quadrilateral with one angle supplementary to both its consecutive angles.]

One angle is supplementary to both its consecutive angles. The quadrilateral is a parallelogram.

### Problem 4

![Diagram of a quadrilateral with one pair of opposite angles congruent.]

One pair of opposite angles are congruent. This is not enough information.

### Problem 5

![Diagram of a quadrilateral with both pairs of opposite angles congruent.]

Both pairs of opposite angles are congruent. The quadrilateral is a parallelogram.

### Problem 6

![Diagram of a quadrilateral with no two pairs of consecutive sides congruent.]

No. Two pairs of consecutive sides congruent does not form a parallelogram.

### Problem 7

Show that quadrilateral $JKLM$ is a parallelogram by using the definition of parallelogram. $J(-1, -6), K(-4, -1), L(4, 5), M(7, 0)$.

<table>
<thead>
<tr>
<th>Side</th>
<th>Slope</th>
</tr>
</thead>
<tbody>
<tr>
<td>$JK$</td>
<td>$m = \frac{-1 - (-6)}{-4 - (-1)} = \frac{5}{-3}$</td>
</tr>
<tr>
<td>$LM$</td>
<td>$m = \frac{0 - 5}{7 - 4} = \frac{-5}{3}$</td>
</tr>
<tr>
<td>$KL$</td>
<td>$m = \frac{5 - (-1)}{4 - (-4)} = \frac{6}{8} = \frac{3}{4}$</td>
</tr>
<tr>
<td>$JM$</td>
<td>$m = \frac{0 - (-6)}{7 - (-1)} = \frac{6}{8} = \frac{3}{4}$</td>
</tr>
</tbody>
</table>

$JK \parallel LM$

$KL \parallel JM$

The quadrilateral has two pairs of parallel sides, therefore it is a parallelogram.

### Problem 8

Show that quadrilateral $ABCD$ is a parallelogram by using Theorem 6-3-1. $A(2, 3), B(6, 2), C(5, 0), D(1, 1)$.

<table>
<thead>
<tr>
<th>Side</th>
<th>Slope</th>
</tr>
</thead>
<tbody>
<tr>
<td>$AB$</td>
<td>$m = \frac{2 - 3}{6 - 2} = \frac{-1}{4}$</td>
</tr>
<tr>
<td>$CD$</td>
<td>$m = \frac{1 - 0}{1 - 5} = \frac{1}{-4}$</td>
</tr>
</tbody>
</table>

$d = \sqrt{(6 - 2)^2 + (2 - 3)^2} = \sqrt{17}$

$d = \sqrt{(1 - 5)^2 + (1 - 0)^2} = \sqrt{17}$

$AB \parallel CD$

$AB = CD$

The quadrilateral has one pair of parallel and congruent sides, therefore it is a parallelogram.
Chapter 6 Study Guide Answers

Geometry – Section 6.4 – Notes and Examples – Properties of Special Parallelograms

A second type of special quadrilateral is a **rectangle**. A rectangle is a quadrilateral with four **right** angles.

<table>
<thead>
<tr>
<th>Theorems</th>
<th>Properties of Rectangles</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>THEOREM</strong></td>
<td><strong>HYPOTHESIS</strong></td>
</tr>
<tr>
<td>6-4-1</td>
<td>If a quadrilateral is a rectangle, then it is a parallelogram. (rect. → □)</td>
</tr>
<tr>
<td>6-4-2</td>
<td>If a parallelogram is a rectangle, then its diagonals are congruent. (rect. → diag. ≅)</td>
</tr>
</tbody>
</table>

Since a rectangle is a **parallelogram** by Theorem 6-4-1, a rectangle “inherits” all the **properties** of parallelograms that you learned in Lesson 6-2.

A **rhombus** is another **special** quadrilateral. A rhombus is a quadrilateral with four **congruent** sides.
Chapter 6 Study Guide Answers

**Theorems Properties of Rhombuses**

<table>
<thead>
<tr>
<th>THEOREM</th>
<th>HYPOTHESIS</th>
<th>CONCLUSION</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.4-3</td>
<td>If a quadrilateral is a rhombus, then it is a parallelogram. (rhombus $\rightarrow$ □)</td>
<td>$ABCD$ is a parallelogram.</td>
</tr>
<tr>
<td>6.4-4</td>
<td>If a parallelogram is a rhombus, then its diagonals are perpendicular. (rhombus $\rightarrow$ diag. $\perp$)</td>
<td>$AC \perp BD$</td>
</tr>
</tbody>
</table>
| 6.4-5   | If a parallelogram is a rhombus, then each diagonal bisects a pair of opposite angles. (rhombus $\rightarrow$ each diag. bisects opp. $\triangle$) | $\angle 1 \equiv \angle 2$  
$\angle 3 \equiv \angle 4$  
$\angle 5 \equiv \angle 6$  
$\angle 7 \equiv \angle 8$ |

Like a __rectangle__, a __rhombus__ is a parallelogram. So you can apply the __properties__ of __parallelograms__ to rhombi.

\[ \begin{array}{c}
\text{B} \\
\text{A} \\
\text{C} \\
\text{D} \\
\end{array} \quad \begin{array}{c}
\text{Square } ABCD \\
\end{array} \]

A __square__ is a quadrilateral with four __right__ angles and four __congruent__ sides. In the exercises, you will show that a square is a __parallelogram__, a __rectangle__, and a __rhombus__. So a square has the properties of all three.

Rectangles, rhombi, and squares are sometimes referred to as __special__ parallelograms.
### Problem 1
A slab of concrete is poured with diagonal spacers. In rectangle \( CNRT \), \( CN = 35 \) and \( NT = 58 \). Find the length of \( TR \) and \( CE \).

\[
\begin{align*}
TR &= 35 \\
CE &= \frac{1}{2}(58) = 29
\end{align*}
\]

### Problem 2
A woodworker constructs a rectangular picture frame so that \( JK = 50 \) and \( JL = 86 \). Find \( HM \).

\[
HM = \frac{1}{2}(86) = 43
\]

### Problem 3
The rectangular gate has diagonal braces. Find \( HJ \) and \( HK \).

\[
\begin{align*}
HJ &= 48 \text{ in.} \\
HK &= JG = 2(30.8) = 61.6 \text{ in.}
\end{align*}
\]

### Problem 4
\( PQRS \) is a rhombus. Find the measure of \( QP \) and \( m\angle QRP \).

\[
\begin{align*}
3a &= 4a - 14 \\
-4a &= -4a \\
-a &= -14 \\
a &= 14 \\
QP &= RS = 3(14) = 42 \\
m\angle QRP &= \frac{1}{2}(180 - 78)° = 51°
\end{align*}
\]

### Problem 5
\( TVWX \) is a rhombus. Find \( TV \) and \( m\angle VTZ \).

\[
\begin{align*}
TV &= XT = 3(1.3) + 4 \\
&= 7.9 \\
WV &= XT = 13b - 9 = 3b + 4 \\
&= 10b + 9 + 9 = 10b + 18 = 13 \\
&= 10b = 13 \\
b &= 1.3 \\
m\angle TZV &= 90° \\
m\angle VTZ &= m\angle XTZ = (5(5) - 5)° = 20°
\end{align*}
\]

### Problem 6
\( CDFG \) is a rhombus. Find \( CD \) and \( m\angle GCH \) if \( m\angle GCD = (b + 3)° \) and \( m\angle CDF = (6b - 40)° \).

\[
\begin{align*}
CG &= GF \\
m\angle GCD + m\angle CDF &= 180° \\
5a &= 3a + 17 \\
-3a &= -3a \\
b + 3 + 6b - 40 &= 180 \\
2a &= 17 \\
a &= 8.5 \\
7b &= 217 \\
7b &= 217 \\
b &= 31 \\
CD &= CG = 5(8.5) = 42.5 \\
m\angle GCH &= \frac{1}{2} m\angle GCD = \frac{1}{2}(31 + 3)° = 17°
\end{align*}
\]
Chapter 6 Study Guide Answers

Problem 7
Show that the diagonals of square \( EFGH \) are congruent perpendicular bisectors of each other.

First we need to find the slopes of the diagonals.

\[
\begin{align*}
    \text{EG} & \quad m = \frac{0-(-1)}{3-(-4)} = \frac{1}{7} \\
    \text{FH} & \quad m = \frac{-4-3}{0-(-1)} = \frac{-7}{1}
\end{align*}
\]

Therefore, \( \text{EG} \parallel \text{FH} \).

Then we need to find the lengths of the diagonals.

\[
\begin{align*}
    d_{\text{EG}} &= \sqrt{(3-(-4))^2 + (0-(-1))^2} = \sqrt{50} \\
    d_{\text{FH}} &= \sqrt{(0-(-1))^2 + (-4-3)^2} = \sqrt{50}
\end{align*}
\]

Therefore, \( \text{EG} = \text{FH} \).

Then we need to find the midpoints of the diagonals.

\[
\begin{align*}
    \left( \frac{-4+3-1+0}{2}, \frac{-1+0}{2} \right) &= \left( \frac{-1}{2}, \frac{1}{2} \right) \\
    \left( \frac{-1+0+3+(-4)}{2}, \frac{-1}{2} \right) &= \left( \frac{-1}{2}, \frac{-1}{2} \right)
\end{align*}
\]

The diagonals are congruent, perpendicular, and have the same midpoint. They are congruent perpendicular bisectors of each other.

Problem 8
The vertices of square \( STVW \) are \( S(-5, -4) \), \( T(0, 2) \), \( V(6, -3) \), and \( W(1, -9) \). Show that the diagonals of square \( STVW \) are congruent perpendicular bisectors of each other.

First we need to find the slopes of the diagonals.

\[
\begin{align*}
    \text{SV} & \quad m = \frac{-3-(-4)}{6-(-5)} = \frac{1}{11} \\
    \text{TW} & \quad m = \frac{-9-2}{1-0} = \frac{-11}{1}
\end{align*}
\]

Therefore, \( \text{SV} \parallel \text{TW} \).

Then we need to find the lengths of the diagonals.

\[
\begin{align*}
    d_{\text{SV}} &= \sqrt{(6-(-5))^2 + (-3-(-4))^2} = \sqrt{122} \\
    d_{\text{TW}} &= \sqrt{(1-0)^2 + (-9-2)^2} = \sqrt{122}
\end{align*}
\]

Therefore, \( \text{SV} = \text{TW} \).

Then we need to find the midpoints of the diagonals.

\[
\begin{align*}
    \left( \frac{-5+6-4+(-3)}{2}, \frac{2}{2} \right) &= \left( \frac{1}{2}, \frac{-7}{2} \right) \\
    \left( \frac{0+1+2+(-9)}{2}, \frac{2}{2} \right) &= \left( \frac{1}{2}, \frac{-7}{2} \right)
\end{align*}
\]

The diagonals are congruent, perpendicular, and have the same midpoint. They are congruent perpendicular bisectors of each other.
Geometry – Section 6.5 – Notes and Examples – Conditions for Special Parallelograms

When you are given a parallelogram with certain properties, you can use the theorems below to determine whether the parallelogram is a rectangle.

**Theorems: Conditions for Rectangles**

<table>
<thead>
<tr>
<th>THEOREM</th>
<th>EXAMPLE</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>6-5-1</strong> If one angle of a parallelogram is a right angle, then the parallelogram is a rectangle. ((\square) with one rt. (\rightarrow) rect.)</td>
<td>![Example of a parallelogram with a right angle]</td>
</tr>
<tr>
<td><strong>6-5-2</strong> If the diagonals of a parallelogram are congruent, then the parallelogram is a rectangle. ((\square) with diags. (\cong) (\rightarrow) rect.)</td>
<td>![Example of a parallelogram with congruent diagonals]</td>
</tr>
</tbody>
</table>

Below are some conditions you can use to determine whether a parallelogram is a rhombus.

**Theorems: Conditions for Rhombuses**

<table>
<thead>
<tr>
<th>THEOREM</th>
<th>EXAMPLE</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>6-5-3</strong> If one pair of consecutive sides of a parallelogram are congruent, then the parallelogram is a rhombus. ((\square) with one pair cons. sides (\cong) (\rightarrow) rhombus)</td>
<td>![Example of a parallelogram with congruent consecutive sides]</td>
</tr>
<tr>
<td><strong>6-5-4</strong> If the diagonals of a parallelogram are perpendicular, then the parallelogram is a rhombus. ((\square) with diags. (\perp) (\rightarrow) rhombus)</td>
<td>![Example of a parallelogram with perpendicular diagonals]</td>
</tr>
<tr>
<td><strong>6-5-5</strong> If one diagonal of a parallelogram bisects a pair of opposite angles, then the parallelogram is a rhombus. ((\square) with diag. bisecting opp. (\angle) (\rightarrow) rhombus)</td>
<td>![Example of a parallelogram with bisected diagonals]</td>
</tr>
</tbody>
</table>

In order to apply Theorems 6-5-1 through 6-5-5, the quadrilateral must be a parallelogram.

To prove that a given quadrilateral is a square, it is sufficient to show that the figure is both a rectangle and a rhombus.
In 1-4, use the diagonals to determine whether a parallelogram with the given vertices is a rectangle, rhombus, or square. Give all the names that apply.

**Problem 1**

\[ P(-1,4), Q(2,6), R(4,3), S(1,1) \]

Find the lengths of the diagonals.

\[ PR = \sqrt{(4-(-1))^2 + (3-4)^2} = \sqrt{26} \]

\[ SQ = \sqrt{(2-1)^2 + (6-1)^2} = \sqrt{26} \]

The diagonals are congruent.

Parallelogram \( PQRS \) is a rectangle.

Find the slopes of the diagonals.

\[ \frac{3-4}{4-(-1)} = -1 \]

\[ \frac{6-1}{2-1} = 1 \]

PR \( \perp \) SQ

The diagonals are perpendicular.

Parallelogram \( PQRS \) is a rhombus.

Since parallelogram \( PQRS \) is both a rhombus and a rectangle, it is a square.

**Problem 2**

\[ W(0,1), X(4,2), Y(3,-2), Z(-1,-3) \]

Find the lengths of the diagonals.

\[ WY = \sqrt{(3-0)^2 + (-2-1)^2} = \sqrt{18} \]

\[ ZY = \sqrt{(4-(-1))^2 + (2-(-3))^2} = \sqrt{50} \]

The diagonals are not congruent.

Parallelogram \( WXYZ \) is not a rectangle.

Find the slopes of the diagonals.

\[ \frac{-2-1}{3-0} = \frac{-3}{3} = -1 \]

\[ \frac{2-(-3)}{4-(-1)} = \frac{5}{5} = 1 \]

WY \( \perp \) ZX

The diagonals are perpendicular.

Parallelogram \( WXYZ \) is a rhombus.

Since \( WXYZ \) is not a rectangle, it is not a square.
A **kite** is a **quadrilateral** with exactly **two** pairs of congruent consecutive sides.

![Kite ABCD]

**Theorems**  
**Properties of Kites**

<table>
<thead>
<tr>
<th>THEOREM</th>
<th>HYPOTHESIS</th>
<th>CONCLUSION</th>
</tr>
</thead>
</table>
| **6-6-1** | If a quadrilateral is a kite, then its diagonals are perpendicular.  
(kite $\rightarrow$ diags. $\perp$) | $\overline{AC} \perp \overline{BD}$ |
| **6-6-2** | If a quadrilateral is a kite, then exactly one pair of opposite angles are congruent.  
(kite $\rightarrow$ one pair opp. $\angle \cong$) | $\angle B \cong \angle D$  
$\angle A \not\cong \angle C$ |

A **trapezoid** is a quadrilateral with exactly **one** pair of parallel sides. Each of the parallel sides is called a **base**. The **nonparallel** sides are called **legs**. Base **angles** of a trapezoid are two **consecutive** angles whose common side is a **base**.
If the legs of a trapezoid are congruent, the trapezoid is an isosceles trapezoid. The following theorems state the properties of an isosceles trapezoid:

<table>
<thead>
<tr>
<th>THEOREM</th>
<th>DIAGRAM</th>
<th>EXAMPLE</th>
</tr>
</thead>
<tbody>
<tr>
<td>6-6-3</td>
<td><img src="image" alt="Diagram" /></td>
<td>$\triangle A \cong \triangle D$, $\triangle B \cong \triangle C$</td>
</tr>
<tr>
<td>6-6-4</td>
<td><img src="image" alt="Diagram" /></td>
<td>$ABCD$ is isosceles.</td>
</tr>
<tr>
<td>6-6-5</td>
<td><img src="image" alt="Diagram" /></td>
<td>$AC \cong DB \iff ABCD$ is isosceles.</td>
</tr>
</tbody>
</table>

The midsegment of a trapezoid is the segment whose endpoints are the midpoints of the legs. In Lesson 5-1, you studied the **Triangle Midsegment Theorem**. The **Trapezoid Midsegment Theorem** is similar to it.

The midsegment of a trapezoid is parallel to each base, and its length is one half the sum of the lengths of the bases.

$$xy = \frac{1}{2}(BC + AD)$$
Chapter 6 Study Guide Answers

**Problem 1**
In kite $ABCD$, $m\angle DAB = 54^\circ$, $m\angle CDF = 52^\circ$, $AB = 20$, and $BC = 12$. Find the measures of $\angle BCD$, $\angle ABC$, and $CD$.

$$m\angle BCD = 2(90 - 52)^\circ = 76^\circ$$
$$m\angle ABC = (90 - 27)^\circ + 52^\circ = 115^\circ$$
$CD = 12$

**Problem 2**
In kite $PQRS$, $m\angle PQR = 78^\circ$, $m\angle TRS = 59^\circ$, $QR = 15$, and $RS = 22$. Find the measures of $\angle QPS$, $\angle PSR$, and $QP$.

$$m\angle QPS = (90 - 39)^\circ + 59^\circ = 110^\circ$$
$$m\angle PSR = 2(90 - 59)^\circ = 62^\circ$$
$QP = 15$

**Problem 3**
Find $m\angle A$.

$$m\angle A = 180^\circ - 100^\circ = 80^\circ$$

**Problem 4**
Find $m\angle F$.

$$m\angle F = 180^\circ - 49^\circ = 131^\circ$$

**Problem 5**
$AD = 12x - 11$, and $BC = 9x - 2$. Find the value of $x$ so that $ABCD$ is isosceles.

$$AD = BC$$
$$12x - 11 = 9x - 2$$
$$-9x____-9x$$
$$3x - 11 = -2$$
$$+11 +11$$
$$3x = 9$$
$$\frac{3x}{3} = \frac{9}{3}$$
$$x = 3$$

**Problem 6**
Find the value of $a$ so that $PQRS$ is isosceles.

$$m\angle S = m\angle P$$
$$2a^2 - 54 = a^2 + 27$$
$$-a^2-a^2$$
$$a^2 - 54 = 27 + 54 + 54$$
$$a^2 = 81$$
$$\sqrt{a^2} = \sqrt{81}$$
$$a = 9$$

**Problem 7**
Find $EF$.

$$\frac{13.5 + 8}{2} = 10.75$$

**Problem 8**
Find $EH$.

$$EH = 16.5 - 8.5 = 8$$